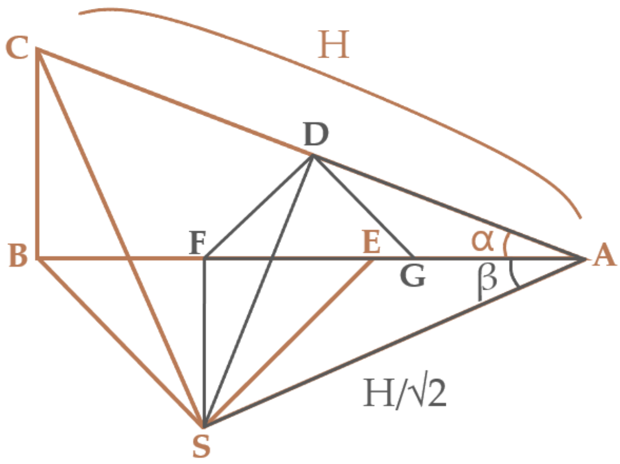
2 new trigonometry formulas in $\pi/4$

Let's consider a first large brown chord figure of angle α drawn adjacent side BA to the horizontal. We place the points F in the middle of BE and D in the middle of CA (the hypotenuse H) and draw a second, smaller grey string figure of angle β embedded in the first and therefore with corresponding lengths between the two figures.



Small figure: FS = GA

Large figure: FS = FE

Calculation of FG: FG = FA – GA = FA – FE = EA = CB (the « chord $\sqrt{2}$ » of the small figure FG is equal to the opposite side CB in the large figure)

Formula for the chord in the small figure:

$$\cos \beta - \sin \beta = FG / SA = CB / (H / \sqrt{2}) = \sqrt{2}\, CB / H = \sqrt{2} \sin \alpha$$

And since it's possible to draw the brown figure in an even larger figure in the same way as the grey one was drawn in the brown one, the writing is reversible:

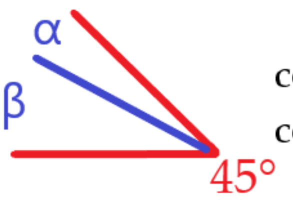
$$\cos \alpha - \sin \alpha = \sqrt{2} \sin \beta$$

Having noted that $\alpha + \beta = 45°$ or $\pi/4$ radians, which makes 2 new trigonometry formulas in $\pi/4$:

$$\cos \alpha - \sin \alpha = \sqrt{2} \sin (\pi/4 - \alpha)$$

$$\cos (\pi/4 - \alpha) - \sin (\pi/4 - \alpha) = \sqrt{2} \sin \alpha$$

Schematic:



$$\alpha + \beta = 45°$$

$$\cos \beta - \sin \beta = \sqrt{2} \sin \alpha$$

$$\cos \alpha - \sin \alpha = \sqrt{2} \sin \beta$$

Registered with the IP on 16/03/2023

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